

Introduction to Statics

.PDF Edition – Version 0.95

Unit 29

Moments of Inertia of Composite Areas

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Unit 29

Moments of Inertia of Composite Areas

Frame 29-1

Introduction

This unit will teach you how to combine the moments of inertia of simple geometric shapes to obtain the moment of inertia of a composite area. It will also teach you how not to combine them.

Moments of inertia of areas are used extensively in "strength" to calculate stresses and deflections in beams.

In American Customary Units we calculate stress in pounds per square inch (psi) so it is the common practice to use areas (and derived properties) measured in inches. Conveniently, most American drawings have areas dimensioned in inches.

In System International (SI) stresses are calculated in Pascals (Newtons per square meter) so it is the common practice to use area areas (and derived properties) measured in meters. For better or worse, cross-sections in most SI engineering drawings are labeled in centimeters or millimeters.

In this unit the "correct" units in answers will be in terms of inches or meters.

The system used for finding the second moment for composite areas is very similar to that used for finding the first moments and centroids of composite areas. It would probably pay you to review your notebook for Unit 12 before beginning the new work. When you have done so, go to the next frame.

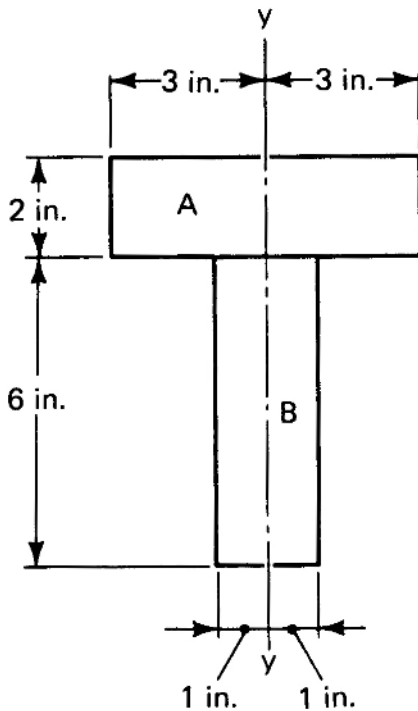
Correct response to preceding frame

No response

Frame 29-2

Common Axis

Second moments of areas may be added directly if the moments of the areas are with respect to the same axis.



For rectangle A

$$I_y = \underline{\hspace{2cm}}$$

For rectangle B

$$I_y = \underline{\hspace{2cm}}$$

For the composite area

$$I_y = \underline{\hspace{2cm}}$$

Express your answers in in^4

Correct response to preceding frame

For A $I_y = \frac{2 \times 6^3}{12} = 36 \text{ in}^4$

For B $I_y = \frac{6 \times 2^3}{12} = 4 \text{ in}^4$

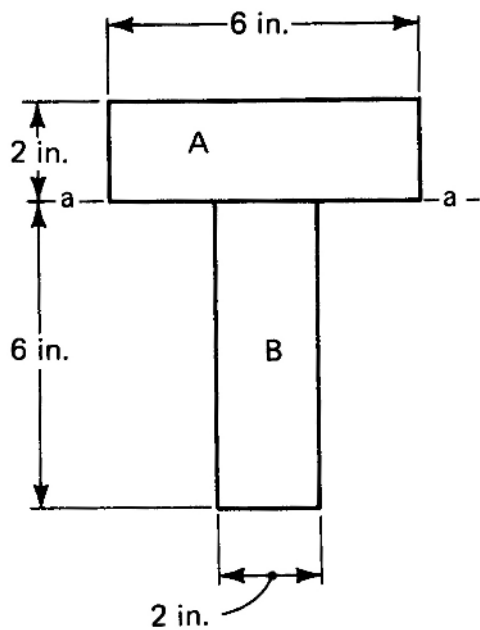
For A + B $I_y = 40 \text{ in}^4$

Frame 29-3

Common Axis

The general expression for the moment of inertia of a rectangle about an edge is

$$I = \frac{b h^3}{3}$$



For part A

$I_{a-a} =$ _____

For part B

$I_{a-a} =$ _____

For the entire area

$I_{a-a} =$ _____

Correct response to preceding frame

For A
$$I_{a-a} = \frac{6 \times 2^3}{3} = 16 \text{ in}^4$$

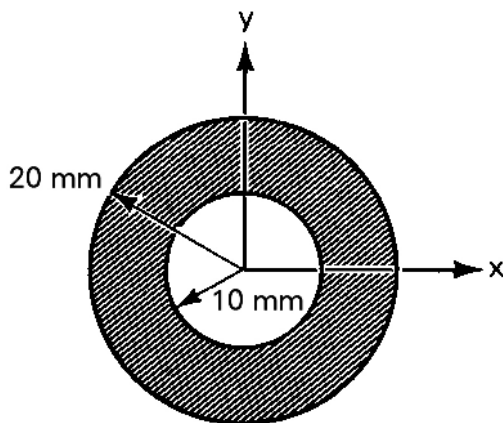
For B
$$I_{a-a} = \frac{2 \times 6^3}{3} = 144 \text{ in}^4$$

For A + B
$$I_{a-a} = 160 \text{ in}^4$$

Frame 29-4

Common Axis

It is also possible to subtract moments of inertia of various areas as long as the moments are taken about the same axis. This allows us to compute the moment of inertia for an area with a hole.



Find the moment of inertia of the hollow circle by computing the following:

For a circle of 20 mm radius

$$I_x = \underline{\hspace{2cm}}$$

For a circle of 10 mm radius

$$I_x = \underline{\hspace{2cm}}$$

For the ring

$$I_x = \underline{\hspace{2cm}}$$

Express your answers in m^4

Correct response to preceding frame

For $r = 20 \text{ mm}$

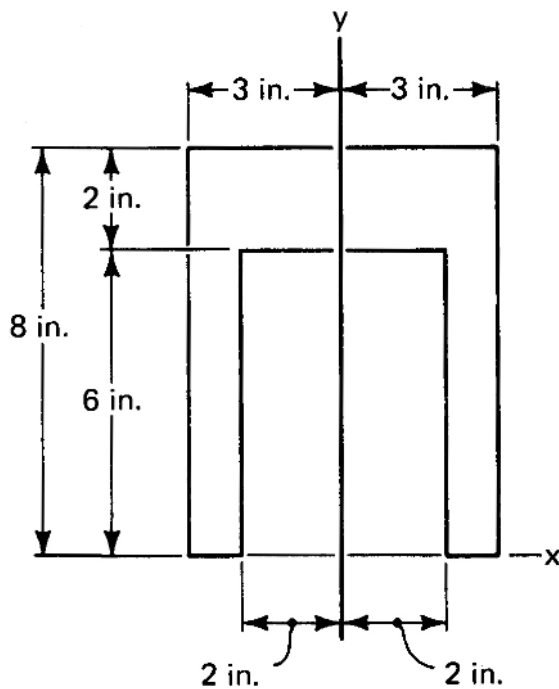
$$I_x = \frac{\pi}{4} \left[20 \times 10^{-3} \text{ m} \right]^4 = \frac{\pi}{4} \left[2 \times 10^{-2} \text{ m} \right]^4 = 4\pi \times 10^{-8} \text{ m}^4$$

For $r = 10 \text{ mm}$ $I_x = \frac{\pi}{4} \left[10 \times 10^{-3} \text{ m} \right]^4 = \frac{\pi}{4} \times 10^{-8} \text{ m}^4$

For hollow circle $I_x = \left[4\pi - \frac{\pi}{4} \right] \times 10^{-8} \text{ m}^4 = \frac{15\pi}{4} \times 10^{-8} \text{ m}^4$

Frame 29-5

Common Axis



Find the moment of inertia of the area shown about the x-axis and about the y-axis.

$I_x =$ _____

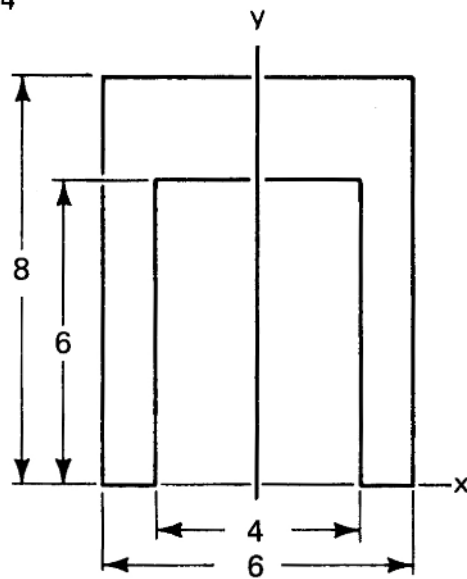
$I_y =$ _____

Correct response to preceding frame

$$I_x = 736 \text{ in}^4$$

Solution:

$$I_y = 112 \text{ in}^4$$



solid rectangle 6 x 8

$$I_y = \frac{8 \times 6^3}{12} = 144 \text{ in}^4$$

$$I_x = \frac{6 \times 8^3}{3} = 1024 \text{ in}^4$$

cut-out 4 x 6

$$I_y = \frac{6 \times 4^3}{12} = 32 \text{ in}^4$$

$$I_x = \frac{4 \times 6^3}{3} = 288 \text{ in}^4$$

composite

$$I_y = 144 - 32 = 112 \text{ in}^4$$

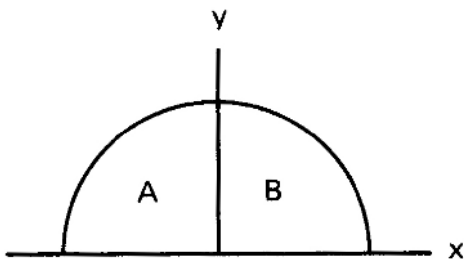
$$I_x = 1024 - 288 = 736 \text{ in}^4$$

Frame 29-6

Common Axis

The moment of inertia of a quarter-circle about its edge is

$$I_x = \frac{\pi r^4}{16}$$



Find I_x and I_y for the semi-circle.

$$I_x = \underline{\hspace{2cm}}$$

$$I_y = \underline{\hspace{2cm}}$$

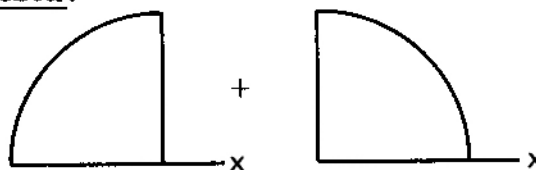
Correct response to preceding frame

$$I_x = \frac{\pi r^4}{8}$$

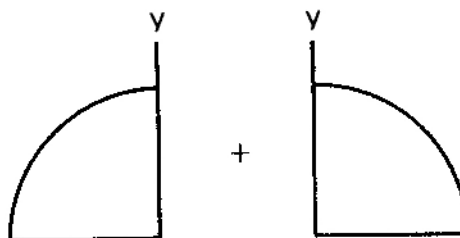
$$I_y = \frac{\pi r^4}{8}$$

Record these important equations on Page 28-4 of your Notebook for later reference.

Solution:



$$I_x = \frac{\pi r^4}{16} + \frac{\pi r^4}{16}$$



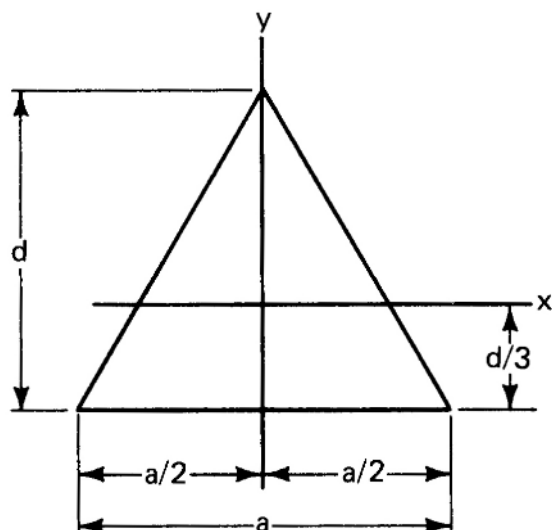
$$I_y = \frac{\pi r^4}{16} + \frac{\pi r^4}{16}$$

The fact that these come out the same is surprising. Nevertheless it is so.

Frame 29-7

Common Axis

The moment of inertia of any triangle may be found by combining the moments of inertia of right triangles about a common axis.



Find I_y for the isosceles triangle shown. Look up I for a triangle in your table if you have forgotten.

$$I_y = \underline{\hspace{2cm}}$$

Correct response to preceding frame

$$I_y = \frac{da^3}{48}$$

Solution:

For a right triangle about an edge

$$I = \frac{bh^3}{12}$$

where h is perpendicular to the axis.

For one right triangle

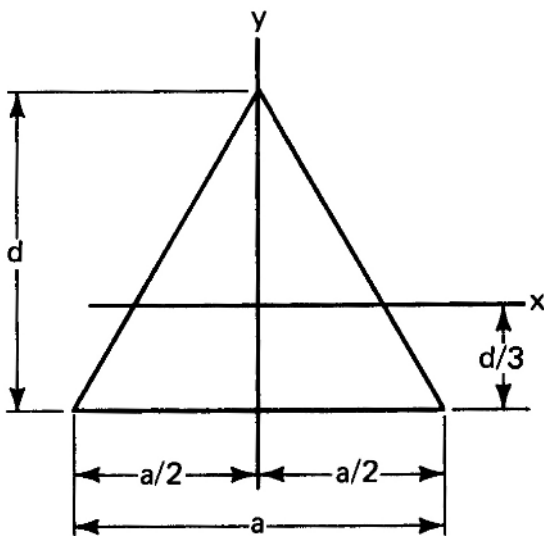
$$I_y = \frac{d \left[\frac{a}{2} \right]^3}{12} = \frac{da^3}{96}$$

For both triangles

$$I_y = \frac{da^3}{48}$$

Frame 29-8

Common Axis



Find I_x for the isosceles triangle shown.

$$I_x = \underline{\hspace{2cm}}$$

Correct response to preceding frame

$$I_x = \frac{ad^3}{36}$$

Solution: For a right triangle about an axis through the centroid and parallel to a base

$$I = \frac{bh^3}{36}$$

where h is perpendicular to the axis.

For one right triangle

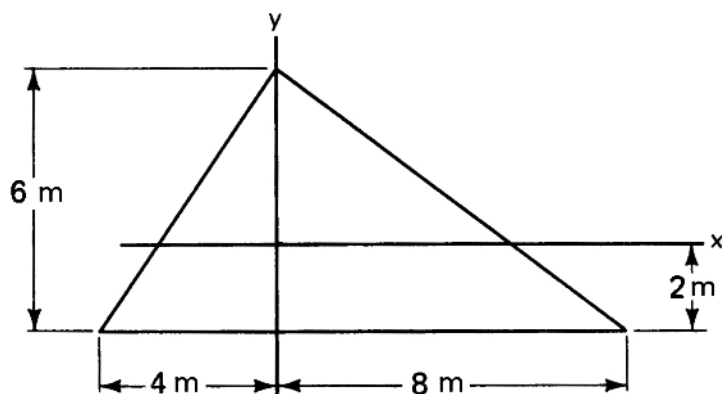
$$I = \frac{\frac{a}{2}d^3}{36}$$

For both right triangles

$$I_x = \frac{ad^3}{36}$$


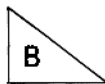
Frame 29-9

Common Axis



Find I_x and I_y for the scalene triangle shown.

In calculating centroids of composite areas I find it useful to use a tabular approach, similar to the one we used with centroids.


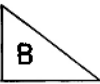
Part	I_{xP}	I_{yP}
		
		
A + B		

Correct response to preceding frame

$$I_x = 72 \text{ m}^4$$

$$I_y = 288 \text{ m}^4$$

Solution:

Part	I_{xP}	I_{yP}
 A	$\frac{4 \times 6^3}{36} = 24$	$\frac{6 \times 4^3}{12} = 32$
 B	$\frac{8 \times 6^3}{36} = 48$	$\frac{6 \times 8^3}{12} = 256$
A + B	72	288

Frame 29-10

Common Axis

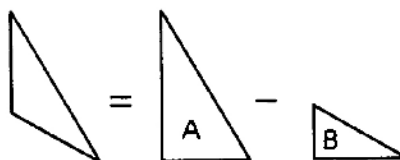
Complete page 29-1 of your notebook.

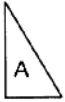

Correct response to preceding frame

$$I_x = 224 \text{ cm}^4 = 224 \times 10^{-8} \text{ m}^4$$

$$I_y = 24 \text{ cm}^4 = 24 \times 10^{-8} \text{ m}^4$$

Solution:



Part	I_{xP}	I_{yP}
	$\frac{6 \times 8^3}{12} = 256$	$\frac{8 \times 6^3}{36} = 48$
	$\frac{6 \times 4^3}{12} = 32$	$\frac{4 \times 6^3}{36} = 24$
A - B	224	24

Frame 29-11

Transition

By now you have probably acquired an excellent grasp of common axis problems.

Alas: It is not always possible to find a single axis about which the moments of inertia of all parts are known. In fact, it can't be done more often than it can, and if you can find such an axis it usually isn't the one you want, so you need to know something more.

The "something more" is usually called the parallel axis theorem and the next group of frames will help you learn it.

Go to the next frame.

Correct response to preceding frame

No response

Frame 29-12

Parallel Axis Theorem

The parallel axis theorem is most simply stated as an equation,

$$I_{xa} = I_{xG} + Ad^2$$

Stated in words it says that the moment of inertia about any axis (I_{xa}) is equal to the sum of the moment of inertia of the area about a parallel axis through its centroid (I_{xG}) plus the product of the area and the square of the distance between (Ad^2 .)

(Comparing the equation and the statement does tend to make one appreciate the equation.)

Go to the next frame.

Correct response to preceding frame

No response

Frame 29-13

Parallel Axis Theorem

The equation

$$I_{xA} = I_{xG} + Ad^2$$

implies that the moment of inertia of an area about an axis passing through the centroid of the area is (***larger*** , ***smaller***) than the moment of inertia about any other parallel axis.

Correct response to preceding frame

smaller

Frame 29-14

Parallel Axis Theorem

The term Ad^2 is often called the "transfer term" and d , the "transfer distance".

To transfer the moment of inertia from the centroidal axis to any other axis the transfer term is (***added*** , ***subtracted***).

Correct response to preceding frame

added

Frame 29-15

Transfer Equation

The Parallel Axis Theorem is, strictly speaking, the word version given in an earlier frame.

The more convenient algebraic form is usually called the "Transfer Equation." Write the transfer equation.

$$I_{xa} = \underline{\hspace{10em}}$$

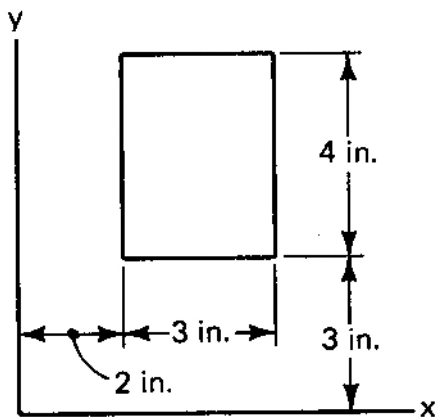
Correct response to preceding frame

$$I_{xa} = I_{xG} + Ad^2$$

Frame 29-16

Parallel Axis Theorem

For the area shown, the centroid is located 3.5 in. from the y axis. So,



$$d = 3.5 \text{ in.}$$

$$I_{yG} = \frac{bd^3}{12} = \frac{4 \times 3^3}{12} = 9$$

$$Ad^2 = 12 \times (3.5)^2 = 147$$

$$I_y = I_{yG} + Ad^2 = 9 + 147 = 156 \text{ in}^4$$

Find I_x

$$I_x = \underline{\hspace{2cm}}$$

Correct response to preceding frame

$$I_x = 316 \text{ in}^4$$

Solution:

$$I_{xG} = \frac{3 \times 4^3}{12} = 16$$

$$d = 5$$

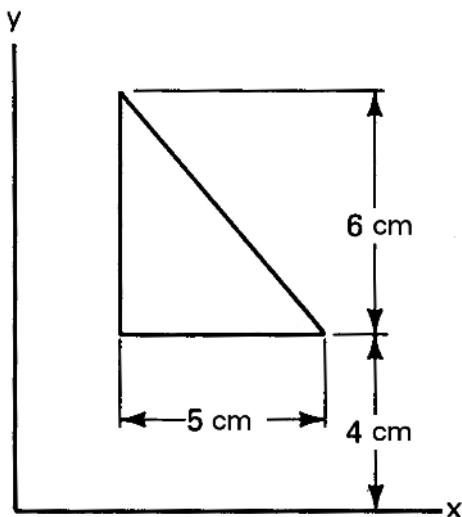
$$Ad^2 = 12 \times 25 = 300$$

$$I_x = 316 \text{ in}^4$$

Frame 29-17

Parallel Axis Theorem

Find I_x for the triangle shown. Begin by drawing in the centroidal x -axis.



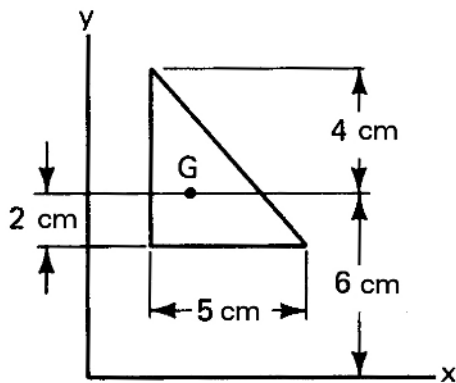
$$d = \underline{\hspace{2cm}}$$

$$Ad^2 = \underline{\hspace{2cm}}$$

$$I_{xG} = \underline{\hspace{2cm}}$$

$$I_x = \underline{\hspace{2cm}}$$

Correct response to preceding frame



$$d = 6 \text{ cm}$$

$$Ad^2 = 15(36) = 540$$

$$I_{xG} = \frac{bd^3}{36} = \frac{5 \times 6^3}{36} = 30$$

$$I_x = 570 \text{ cm}^4 = 570 \times 10^{-8} \text{ m}^4$$

Frame 29-18

Transfer Equation

It is also possible to use the transfer equation in reverse. To do so you must know the moment of inertia about a non-centroidal axis and want to find it about the centroidal axis.

In this case the equation for I_{xG} is

$$I_{xG} = \underline{\hspace{2cm}}$$

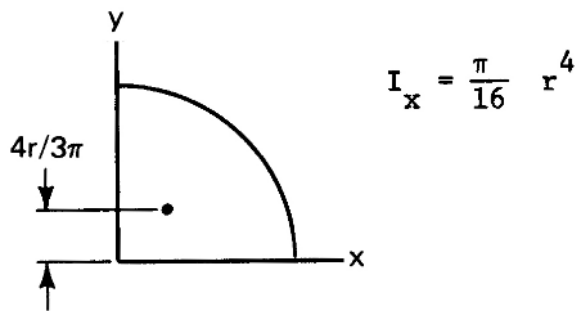
Correct response to preceding frame

$$I_{xG} = I_x - Ad^2$$

Frame 29-19

Transfer Equation

Find I_{xG} for the quarter circle.



$$I_{xG} = \underline{\hspace{10em}}$$

Correct response to preceding frame

$$I_{xG} = .055r^4$$

Solution:

$$d = \frac{4r}{3\pi}$$

$$Ad^2 = \frac{\pi r^2}{4} \left(\frac{4r}{3\pi} \right)^2 = \frac{4r^4}{9\pi}$$

$$I_{xG} = \frac{\pi r^4}{16} - \frac{4r^4}{9\pi}$$

Frame 29-20

Notebook

Do page 29-2 in your notebook. Then complete the Properties of Areas Table on page 28-4 in your notebook.

Correct response to preceding frame

Problem 29-2

$$I_y = 148\pi \text{ in.}^4$$

Problem 29-3

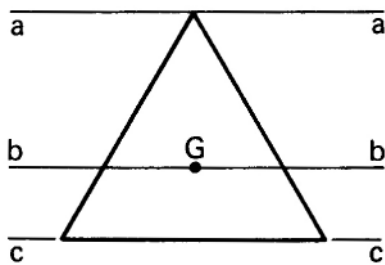
$$\begin{aligned} I_{xG} &= \frac{\pi r^4}{8} - \frac{\pi r^2}{2} \frac{16r^2}{9\pi} \\ &= \frac{\pi r^4}{8} - \frac{8r^4}{9\pi} = .11r^4 \end{aligned}$$

Frame 29-21

Limitation

There is one important limitation on the use of the transfer equation. One of the moments of inertia involved must be centroidal. It is impossible to transfer directly from one non-centroidal axis to another.

In each of the following tell whether the transfer equation applies.



1. Axis a-a to axis b-b

☐

Yes

☐

No

2. Axis b-b to axis c-c

☐

Yes

☐

No

3. Axis a-a to axis c-c

☐

Yes

☐

No

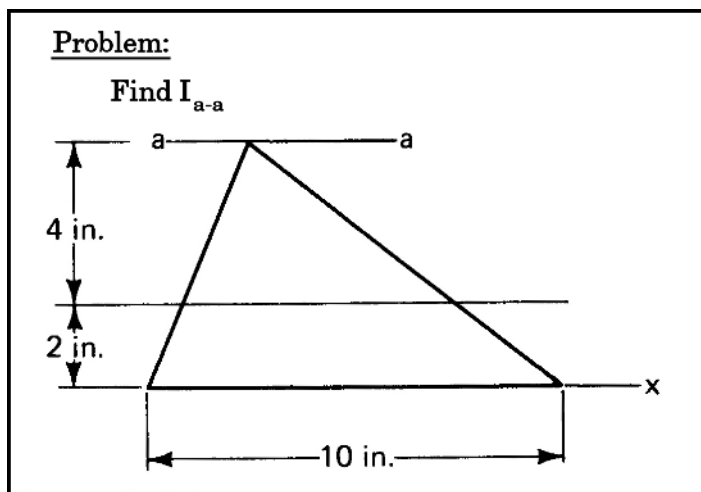
Correct response to preceding frame

1. Yes
2. Yes
3. No

Frame 29-22

Transfer Equation

Of course it is possible to go from any axis to any axis by an indirect transfer.



For the triangle shown:

$$I_x = 180 \text{ in}^4$$

First find I_G

$$I_G = \underline{\hspace{2cm}}$$

Now find I_{a-a}

$$I_{a-a} = \underline{\hspace{2cm}}$$

Correct response to preceding frame

$$I_{xG} = I_x - Ad_1^2$$

$$I_{xG} = 180 - 30(2)^2 = 60 \text{ in.}^4$$

$$I_{a-a} = I_{xG} + Ad_2^2 = 60 + 30(4)^2 = 540 \text{ in.}^4$$

All of which is rather silly since most people would simply begin by finding I_{xG} . However, it can be done this way if one really needs to.

Frame 29-23

Transition

Now you have all the necessary tools for finding moments of inertia of composite areas. All that remains is to learn to use them on composite areas.

The remainder of this unit will be devoted to some pretty complex areas and a handy method for cutting the computations down to size.

This is a good place for a break. I suggest you take one before turning the page.

Correct response to preceding frame

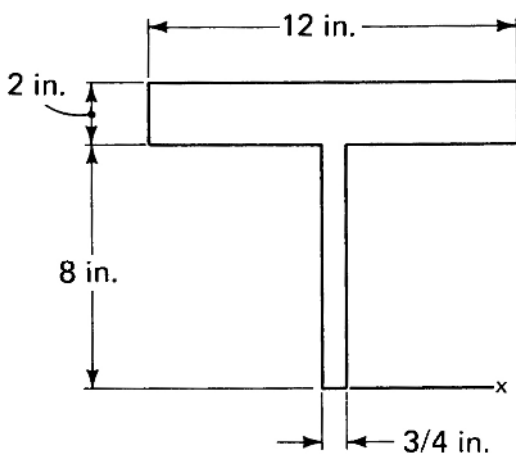
No response


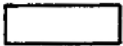
Frame 29-24

Composite Areas by Transfer

The moment of inertia of a composite area about any axis may be found by finding the moments of inertia of all parts about the axis by means of the transfer equation and then adding them.



It is usually a good idea to do this by means of a table. As in Unit 12 the subscript "P" means for the part. Complete the computation for finding I_x for the area shown.



Part	A_P	y_{GP}	$A_P y_{GP}^2$	I_{xGP}	I_{xP}
	6	4	96		
					

$I_x =$ _____

Correct response to preceding frame

Part	A_P	y_{GP}	$A_P y_{GP}^2$	I_{xGP}	I_{xP}
	6	4	96	32	128
	24	9	1944	8	1952

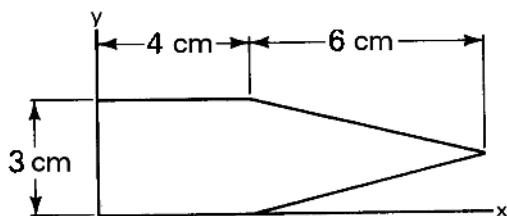
$$I_x = 2080 \text{ in}^4$$

Frame 29-25

Composite Areas by Transfer

The most important single point to remember is that I_{GP} in the table means moment of inertia of the part about its own centroid. Do not use any other moment of inertia in this column.

Find I_y for the area shown.






Part	A_P	x_{GP}	$A_P x_{GP}^2$	I_{yGP}	I_{yP}

$$I_y = \underline{\hspace{2cm}}$$

Correct response to preceding frame

$$I_y = 406 \text{ cm}^4$$
$$= 406 \times 10^{-8} \text{ m}^4$$

Solution:

Part	A_P	x_{GP}	$A_P x_{GP}^2$	I_{yGP}	I_{yP}
	12	2	48	16	64
	4.5	6	162	9	171
	4.5	6	162	9	171

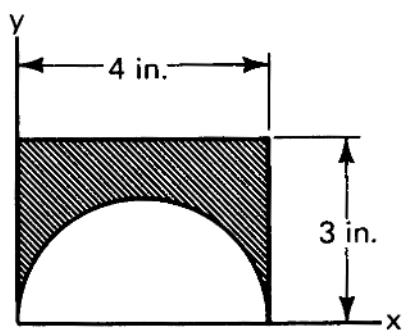
$$I_y = 406$$


Frame 29-26

Holes

Some areas have holes in them. A hole has a negative area and a negative moment of inertia.

Find I_y for the area shown





Part	A_P	x_{GP}	$A_P x_{GP}^2$	I_{yGP}	I_{yP}
	-2π			-2π	

$I_y =$ _____

Correct response to preceding frame

$$I_y = 32.6 \text{ in}^4$$

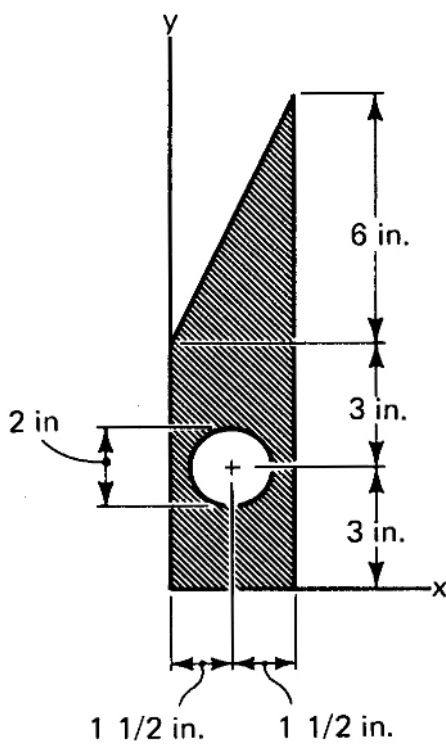
Solution:

Part	A_P	x_{GP}	$A_P x_{GP}^2$	I_{yGP}	I_{yP}
	12	2	48	16	64
	-2π	2	-8π	-2π	-31.4

$$I_y = 32.6$$

Frame 29-27

Composite Areas






Find I_x for the area shown.

This time I'll leave drawing the table to you.

Correct response to preceding frame

$$I_x = 781 \text{ in}^4$$

Solution:

Part	A_P	y_{GP}	$A_P y_{GP}^2$	I_{xGP}	I_{xP}
	9	8	576	18	594
	18	3	162	54	216
	$-\pi$	3	-9π	$-\frac{\pi}{4}$	-29.1

$$I_x = 781$$

Frame 29-28

Transition

Now we come to the reason for all the work you have done on composite areas. You will find in a later course that the strength of a beam is directly related to the moment of inertia of its cross-section about a centroidal axis. Most beams used for heavy loads have composite cross-sections, so there you are.

You can now find the moment of inertia of a composite area about a specified axis. However you need to find it about a centroidal axis. The transfer gives no trouble if you know where the centroid is, but you must usually locate the centroid.

In other words, if you have not yet reviewed Unit 12, please do so now. Then go to the next frame.

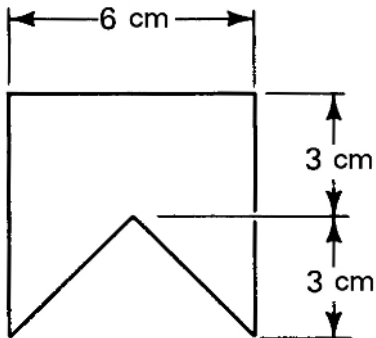
Correct response to preceding frame

No response

Frame 29-29

Review




Set up a table and calculate the y-coordinate of the centroid of the area shown below.



Correct response to preceding frame

$$y_G = \frac{99}{27} = 3.67 \text{ cm}$$

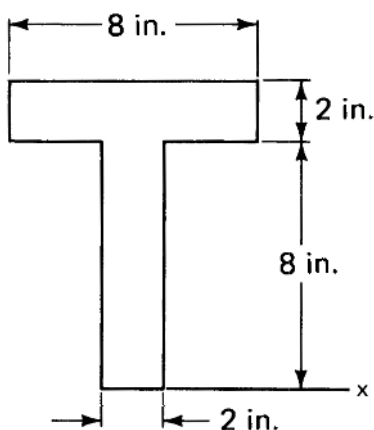
Solution:

Part	A_P	y_{GP}	$A_P y_{GP}$
	36	3	108
	-9	1	-9
Total	27		99

Frame 29-30

Centroid and Moment of Inertia




It is possible to find the centroid and moment of inertia from the same table by adding just one column to the moment of inertia table and finding two more totals, as shown below.



What is the heading of the new column? _____

What are the additional totals? _____

Use the table to find I_x and the y coordinate of the centroid for the area shown.

Part	A_P	y_{GP}		$A_P y_{GP}^2$	I_{xGP}	I_{xP}
Total						

Correct response to preceding frame






Add column $A_P y_{GP}$

Sum columns A_P and $A_P y_{GP}$

$$I_x = 1640 \text{ in}^4$$

$$y_G = \frac{208}{32} = 6.5 \text{ in}$$

Solution:

Part	A_P	y_{GP}	$A_P y_{GP}$	$A_P y_{GP}^2$	I_{xGP}	I_{xP}
	16	9	144	1296	5.33	1301.3
	16	4	64	256	85.33	341.3
Total	32		208			1642.6

Frame 29-31

Centroidal Moment of Inertia

Now make use of the information from the solution above to find I_{xG} . Use the transfer equation in the form $I_{xG} = I_x - Ad^2$

$$d = y_G = \underline{\hspace{2cm}}$$

$$I_x = \underline{\hspace{2cm}}$$

$$Ad^2 = \underline{\hspace{2cm}}$$

$$I_{xG} = \underline{\hspace{2cm}}$$

Correct response to preceding frame

$$y_G = 6.5$$

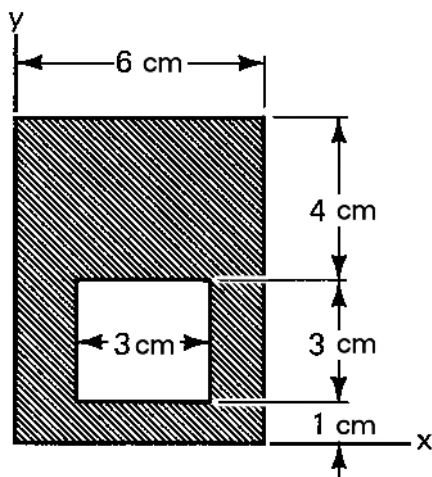
$$I_x = 1643$$

$$Ad^2 = 1352$$

$$I_{xG} = 288 \text{ in}^4$$

Frame 29-32

Centroidal Moment of Inertia



Find I_{xG} for the area shown. (The hole is handled as a negative area and a negative moment of inertia just as you have done before.)

Part	A_P	y_{GP}	$A_P y_{GP}$	$A_P y_{GP}^2$	I_{xGP}	I_{xP}
Total		X		X	X	

$$y_G = \underline{\hspace{2cm}}$$

$$I_x = \underline{\hspace{2cm}}$$

$$Ad^2 = \underline{\hspace{2cm}}$$

$$I_{xG} = \underline{\hspace{2cm}}$$

Correct response to preceding frame

$$y_G = 4.35 \text{ cm}$$



$$I_x = 961 \text{ cm}^4$$

$$Ad^2 = 737 \text{ cm}^4$$

$$I_{xG} = 224 \text{ cm}^4$$

$$= 224 \times 10^{-8} \text{ m}^4$$

Solution:

Part	A_P	y_{GP}	$A_P y_{GP}$	$A_P y_{GP}^2$	I_{xGP}	I_{xP}
	48	4	192	768	256	1024
	-9	2.5	-22.5	-56.25	-6.75	-63
Total	39	X	169.5	X	X	961

Frame 29-33

Transition

The axis about which the moment of inertia is first found is called the "reference axis." In the problems you have worked so far, your selection of a reference axis has been guided. In most problems you will find that no such guidance is provided and that a poor choice of reference axis makes a problem unnecessarily hard.

Consequently the next section of the unit will be devoted to helping you avoid poor choices and make good ones.

Go to the next frame.

Correct response to preceding frame

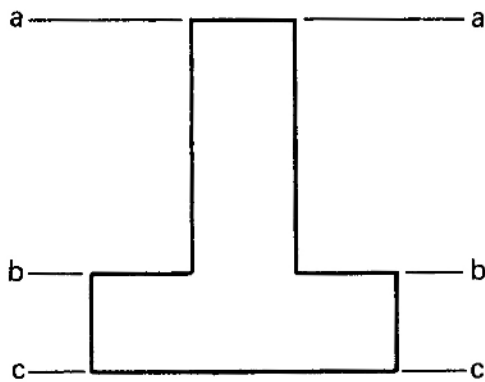
No response

Frame 29-34

Selection of Reference Axis

It is frequently wise to choose the reference axis at one edge of the area so that negative values for centroidal distances are avoided. However, this will often result in rather large numbers in the AyPG column which may be a disadvantage if you don't have a calculating device at hand.

Which reference axis would you use for the area below?



☐ a-a

☐ b-b

☐ c-c

Correct response to preceding frame

My answer is **b-b** or **c-c**.

Not **a-a**. It introduces large numbers and negative coordinates.

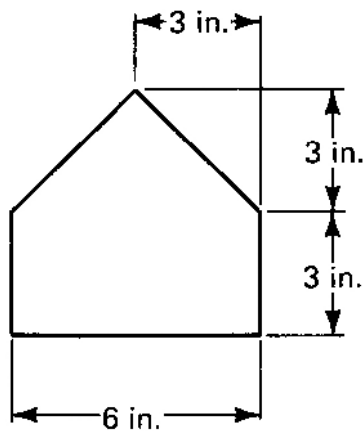
Axis **b-b** will give you easy numbers but a negative coordinate. This choice is particularly good if you remember I for an edge axis of a rectangle.

Axis **c-c** will give you all positive coordinates but larger numbers.

Frame 29-35

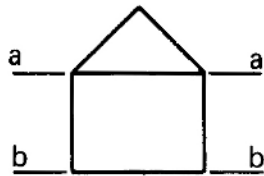
Choice of Axis

Choose your reference axis and find I_{xG} for the area shown.








Correct response to preceding frame

$$I_{xG} = 55.5 \text{ in}^4$$



Axes a-a and b-b are both good choices.

Solution: (Using a-a)

Part	A_P	y_{GP}	$A_P y_{GP}$	$A_P y_{GP}^2$	I_{xGP}	I_{xP}
	9	1	9	9	$\frac{6(3)^3}{36}$	13.5
	18	-1.5	-27	40.5	$\frac{6(3)^3}{12}$	54
Total	27		-18			67.5

$$I_{xG} = 67.5 - 27 \left[\frac{-18}{27} \right]^2$$

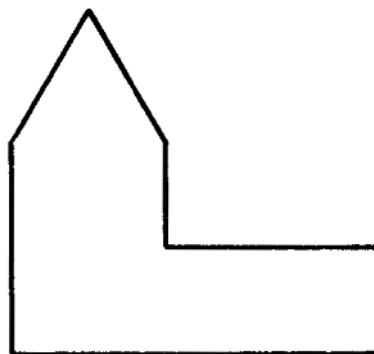
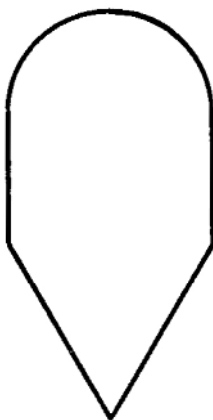
Frame 29-36

Choice of Reference Axis

A reference axis which passes through the centroid of the entire area is ideal if one is available.

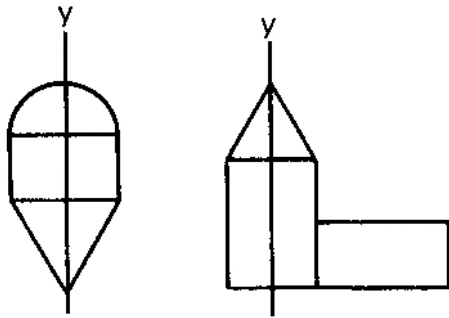
Another good choice is a reference axis which passes through the centroids of several parts.

Both of these choices will make many terms in the table equal zero. Choose reference axes for finding I_{yG} for the areas below. Show them on the figures. Also show how you would divide the areas.



Correct response to preceding frame

My choices would be those shown below.



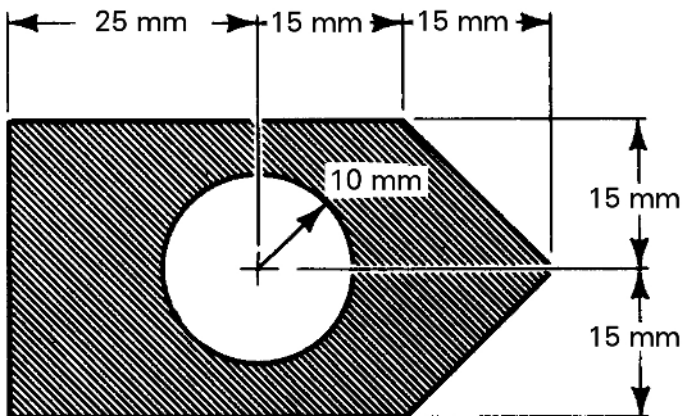
Other good choices and divisions could be made for the second area.

Frame 29-37

Choice of References Axes

A fourth good choice is an axis passing through the center of the one most difficult part to compute. This is particularly applicable to circles and reduces the number of π terms you must compute.

Select an axis and find I_{yG} for the area shown. Draw your own table.

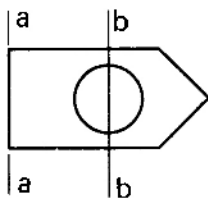


Correct response to preceding frame

$$I_{yG} = 273000 \text{ mm}^4$$

$$= 2.73 \times 10^{-3} \text{ m}^4$$

Solution: (Using b-b)



Either a-a or b-b
is a fair choice
but I would use b-b.

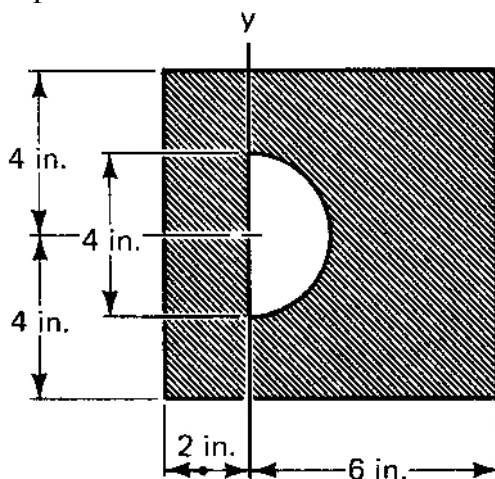
Part	A_P	x_{GP}	$A_P x_{GP}$	$A_P x_{GP}^2$	I_{yGP}	I_{yP}
	1200	-5	-6000	30000	$\frac{30(40)^3}{12}$	190000
	225	20	4500	90000	$\frac{30(15)^3}{36}$	92813
	-314	0	0	0	$-\frac{\pi(10)^4}{4}$	-7854
Total	1111	X	-1500	X	X	274959

$$I_{yG} = 274959 - 1111 \left[\frac{-1500}{1111} \right]^2$$

Frame 29-38

Choice of Reference Axes

The last good choice is an axis about which you know the moment of inertia of several parts. This one is very risky for a beginner but is probably worth the risk if you must cope with a semi-circle.





Choose the y-y axis as a reference axis for the problem shown and complete the table to find I_{yG} .

Part	A_P	x_{GP}	$A_P x_{GP}$	$A_P x_{GP}^2$	I_{yGP}	I_{yP}
Total		X		X	X	

Correct response to preceding frame

$$I_{yG} = 329 \text{ in.}^4$$

Part	A_P	x_{GP}	$A_P x_{GP}$	$A_P x_{GP}^2$	I_{yGP}	I_{yP}
	64	2	128	256	341.3	597.3
	-2π	$\frac{4(2)}{3\pi}$	-5.33	X	X	-6.28
Total	57.7	X	123	X	X	591

$$x_G = \frac{123}{57.7} = 2.13$$

$$I_{yG} = I_y - Ad^2 = 591 - 262$$

$$(\text{For semi-circle } I_{yP} = \frac{\pi}{8} r^4 = \frac{\pi}{8} \times 16 = 2\pi)$$

Frame 29-39

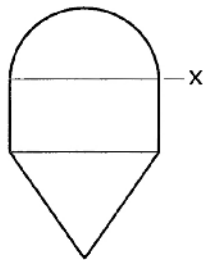
Notebook

Complete page 29-3 of your notebook.




Correct response to preceding frame

Problem 29-4

$$I_{xG} = 846 \text{ in.}^4$$



Solution:

Part	A_P	y_{GP}	$A_P y_{GP}$	$A_P y_{GP}^2$	I_{xGP}	I_{xP}
	25.13	1.698	42.66	XXXX	XXXX	$\frac{\pi(4)^4}{8}$
	32	-2	-64	128	42.67	170.7
	24	-6	-144	864	48	912
Total	81.13	XXXX	-165.3	XXXX	XXXX	1183

$$y_G = -2.04 \text{ in.}$$

$$I_{xG} = 1183 - 81.13 \left[\frac{-165.3}{81.13} \right]^2$$

Frame 29-40

Closure

There are many short-cuts for finding moments of inertia of areas which we have not taught you. If you do much work of this sort you will develop your own tricks.

Slick tricks and short-cuts are for the pro. The amateur, and you will retain that status for many more problems, only gets lost in his own cleverness and ends up with the wrong answer. For some time to come your best policy will be to construct the table and fill it out carefully. If you do that your success is almost guaranteed.